EPFL

# Wire antennas

Antennas

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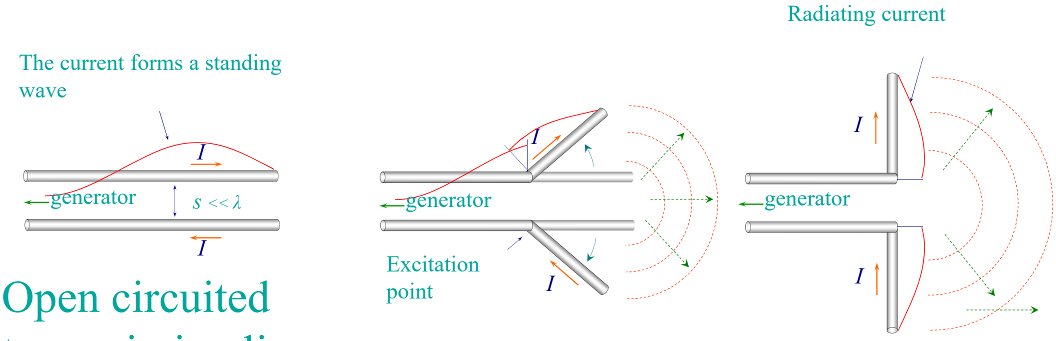
## content

- Dipoles
- Loops

## Wire antenna (1)

The current is obtained form transmission line theory




Open circuited transmission line

($s << \lambda$) no radiation

Starts to radiate

Dipole antenna connected to a transmission line

Max radiation

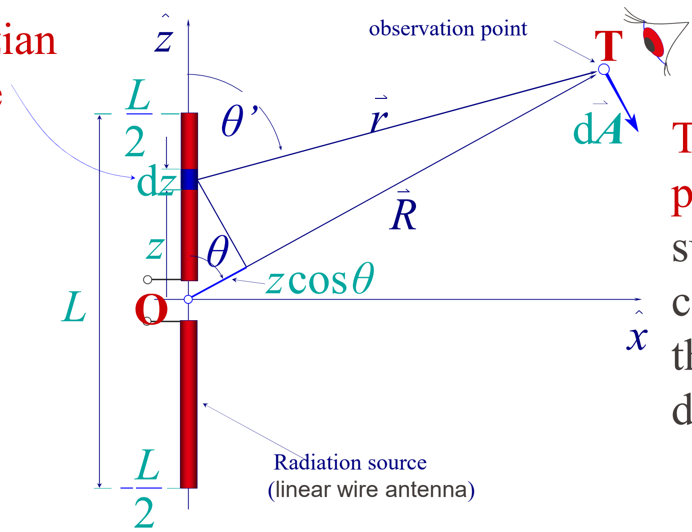
Courtesy: Prof. J. Bartolic, university of Zagreb



## Wire antenna (2)

Geometry of the problem

Hertzian dipole

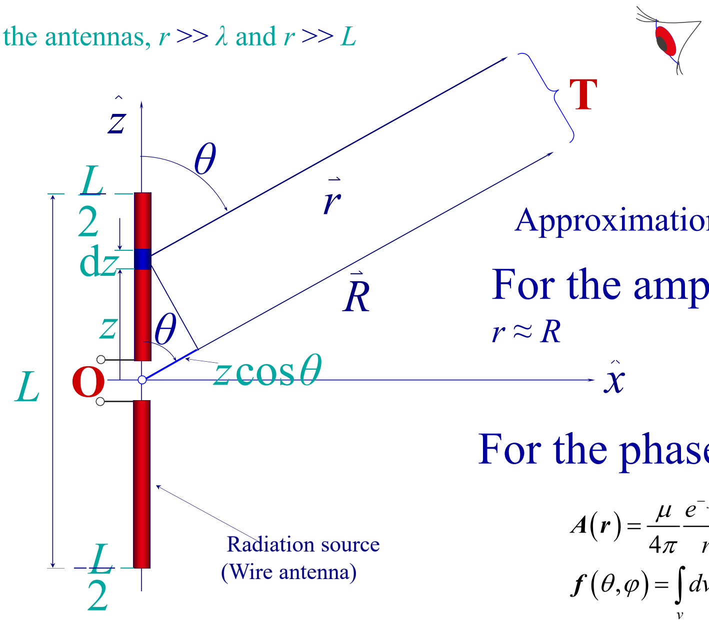



Total vector potential: superposition of the contribution of all the elementary dipoles

Courtesy: Prof. J. Bartolic, université de Zagreb

# Wire antenna (3)

Far from the antennas, $r >> \lambda$ and $r >> L$




Approximation :

For the amplitude: $r \approx R$

For the phase: $r \approx R - z\cos\theta$

$$A(r) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r} f(\theta,\varphi)$$

$$f(\theta,\varphi) = \int_v dv' J(r') e^{jk e_r \cdot r'}$$

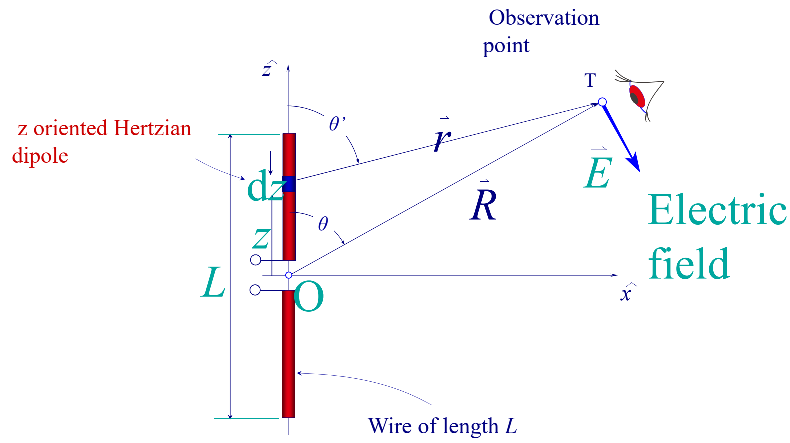


Courtesy: Prof. J. Bartolic, university de Zagreb



# Dipole antenna

- Fields
- Patterns
- Polarization
- Directivity
- Impedance
- Effective aperture

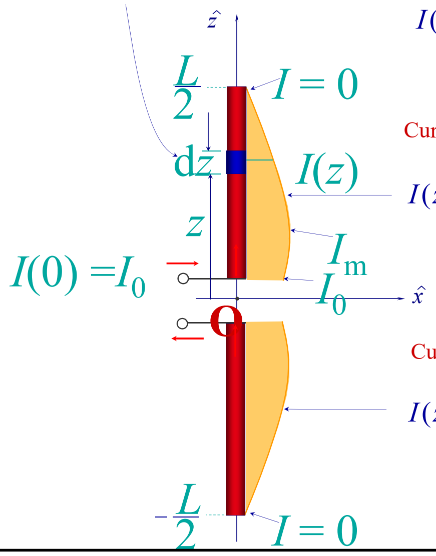





Courtesy: Prof. J. Bartolic, university de Zagreb

## Half wave dipole

Courtesy: Prof. J. Bartolic, university de Zagreb




### Current distribution

$$I(z) = I_m \sin\left[\beta\left(\frac{L}{2} - |z|\right)\right]$$

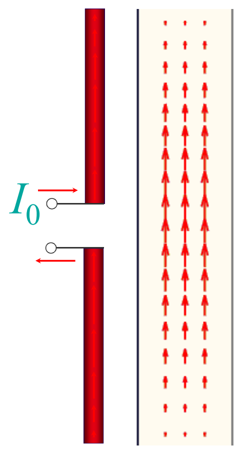
Current on the upper half

$$I(z) = I_m \sin\left[\beta\left(\frac{L}{2} - z\right)\right]$$

Current on the lower half

$$I(z) = I_m \sin\left[\beta\left(\frac{L}{2} + z\right)\right]$$

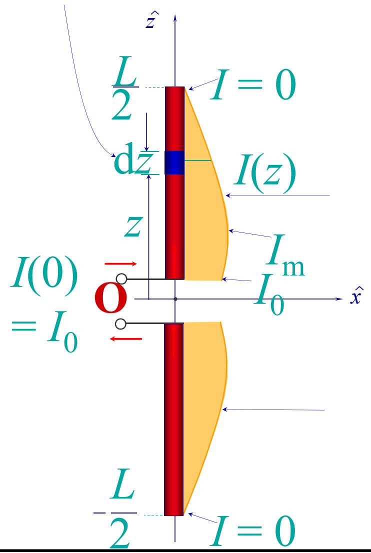
### Vector distribution






## Half wave dipole

Courtesy: Prof. J. Bartolic, université de Zagreb




### Current distribution on the wire

$$\boldsymbol{f} = \hat{\mathbf{z}} \int_{-L/2}^{L/2} I(z')\, e^{jkz' \cos\theta} dz'$$

$$\text{f} = \hat{\mathbf{z}}\, I_{max} \left\{ \int_0^{L/2} \sin[k(L/2 - z')]\, e^{+jkz' \cos\theta} dz' + \int_{-L/2}^{0} \sin[k(L/2 + z')]\, e^{+jkz' \cos\theta} dz' \right\}$$

$$\boldsymbol{f} = \hat{\mathbf{z}}\, 2I_{max} \int_0^{L/2} \sin[k(L/2 - z')] \cos(kz' \cos\theta)\, dz'$$

$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\boldsymbol{f} = \hat{\mathbf{z}} \frac{2I_{max}}{k} \left[\frac{\cos(k\,L/2 \cos\theta) - \cos(k\,L/2)}{\sin^2\theta}\right]$$

# Dipole antenna

$$\boldsymbol{E} = E_0\hat{\boldsymbol{\theta}}$$
$$= jZ_c I_{max}\frac{e^{-jkr}}{2\pi r}\left[\frac{\cos(k\,L/2\cos\theta)-\cos(k\,L/2)}{\sin\theta}\right]\hat{\boldsymbol{\theta}}$$
$$= j60I_{max}e^{-jkr}\left[\frac{\cos(k\,L/2\cos\theta)-\cos(k\,L/2)}{\sin\theta}\right]\hat{\boldsymbol{\theta}}$$

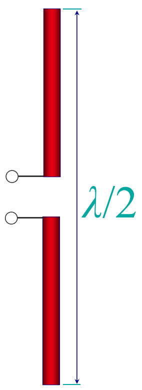


# Half wave dipole

Courtesy: Prof. J. Bartolic, university of Zagreb

$L = \lambda/2$

Amplitude of the E Field




$$E_\theta = \frac{60I_m}{R}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)-\cos\left(\frac{\pi}{2}\right)}{\sin\theta}$$

$$E_\theta = \frac{60I_m}{R}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

Radial dependency

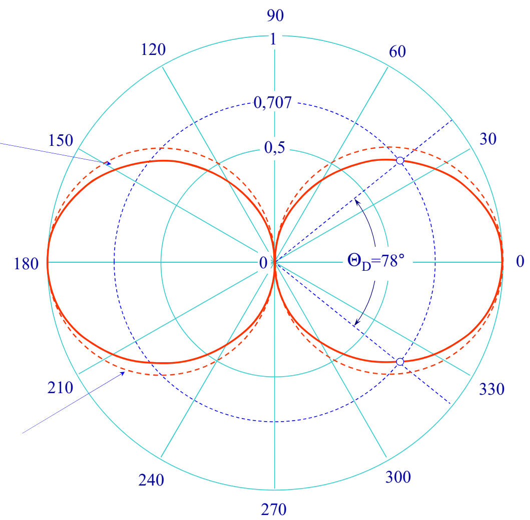
Radiation pattern $F(\theta)$

## Radiation pattern of the half wave dipole (1)

$$L = \lambda/2$$

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$$F(\theta) \approx \sin^{\frac{4}{3}}\theta$$




Radiation pattern of the Hertzian dipole
$F(\theta) = \sin\theta$

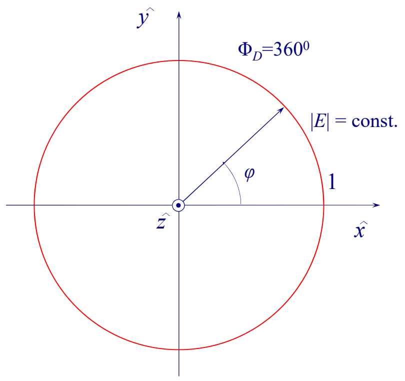
Courtesy: Prof. J. Bartolic, university of Zagreb



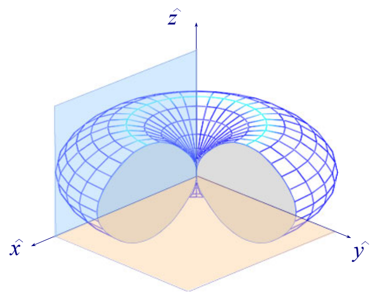
## Radiation pattern of the half wave dipole (2)

$$L = \lambda/2$$

Cut at θ=90°




3 D pattern




Courtesy: Prof. J. Bartolic, university of Zagreb

## Half wave dipole: radiated power

Courtesy: Prof. J. Bartolic, university of Zagreb

$$L = \lambda/2$$

The total radiated power of the dipole can be obtained by integrating the average value of the Poynting vector ***S*** over a sphere of radius *r* surrounding the dipole.

$$P_Z = \oint_S \langle \vec{S}_r \rangle \hat{n} dS = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \frac{|\vec{E}|^2}{2\eta} r^2 \sin(\theta) d\varphi d\theta =$$

$$= 73{,}13 I_m^2 = 73{,}13 I_0^2$$

To obtain this we used $C_{in}(2\pi) = \int_0^{2\pi} \left( \frac{1 - \cos y}{y} \right) dy \approx 2.435$

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## Input impedance

$$L = \lambda/2$$

A detailed analysis shows that along with the radiation resistance, the infinitely thin half wave dipole also exhibits an inductive reactance of 42.5 Ω. This reactance is a consequence of the energy stored in the nearby dipole fields.

Thus, the imedance of the half wave dipole is given by

$$\mathbf{Z_A = 73{,}13 + j\ 42{,}5\ \Omega}$$

Resistance Reactance

The positive value of reactance refers to the fact that the stored energy is magnetic

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## Effective lengthe of a half wave dipole

$$L = \lambda/2$$

The effective length is defined as follows

$$l_{\mathrm{ef}} = \frac{1}{I(0)} \int_{-\frac{L}{2}}^{\frac{L}{2}} I(z)\,\mathrm{d}z$$

where $I(0) = I_m$ Thus:

$$l_{\mathrm{ef}} = \frac{1}{I(0)} \int_{-\frac{L}{2}}^{\frac{L}{2}} I_{\mathrm{m}} \sin\left[\beta\left(\frac{L}{2} - |z|\right)\right] \mathrm{d}z = 2\int_{0}^{\lambda/4} \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{4} - z\right)\right] \mathrm{d}z$$

$$l_{\mathrm{ef}} = 2\int_{0}^{\lambda/4} \sin\left(\frac{\pi}{2} - \frac{2\pi}{\lambda} z\right) \mathrm{d}z = 2\int_{0}^{\lambda/4} \cos\left(\frac{2\pi}{\lambda} z\right) \mathrm{d}z = \frac{2}{2\pi/\lambda} \sin\left(\frac{2\pi}{\lambda} z\right)\Bigg|_{0}^{\lambda/4} = \frac{\lambda}{\pi}$$

*Finally, the effective length of a half wave dipole is given by*

$$l_{\mathrm{ef}} = \frac{\lambda}{\pi} \approx \frac{\lambda}{3}$$



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## Directivity of a half wave dipole

$$L = \lambda/2$$

$$D_0 = 4\pi \frac{U_{\max}}{P_{rad}} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}} = \frac{4}{C_{in}(2\pi)} \simeq 1.643$$

$$10 \log D \approx 2{,}15\,\mathrm{dB}$$

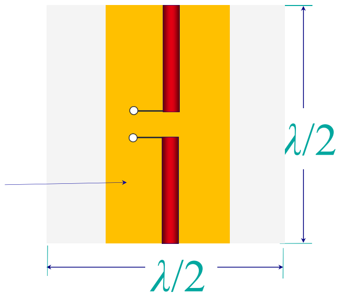
## Effective aperture of a half wave dipole

Directiviy and effective aperture are linked by

$$D = \frac{4\pi}{\lambda^2} A_{ef}$$

Thus

$$A_{ef} = \frac{\lambda^2}{4\pi} D = 0.13\lambda^2$$



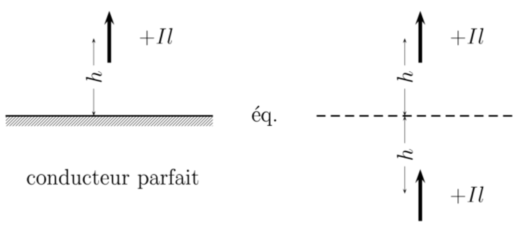



Courtesy: Prof. J. Bartolic, university of Zagreb



## Image theory

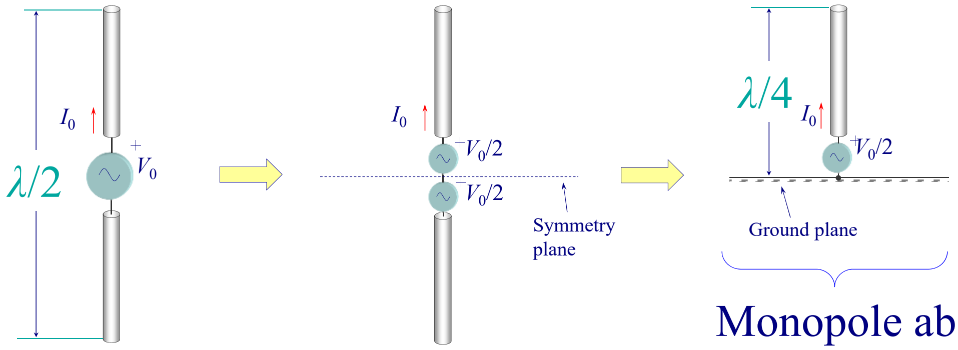
How to consider an antenna above a ground plane



# The monopole antenna(1)

Courtesy: Prof. J. Bartolic, university of Zagreb

Instead of a half-wave dipole, a quarter-wave monopole can be used above a perfect ground plane. By reflection, a half-wave dipole is obtained, so most of the radiation characteristics of the monopole are similar to those of the dipole.




Monopole above ground plane

Hence the radiation resistance is $Z_{unipol} = \frac{V_0}{2 I_0} = \frac{Z_{dipol}}{2} = \frac{73\ \Omega}{2} = 36{,}5\ \Omega$

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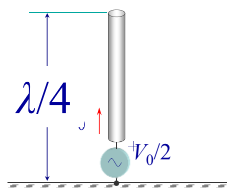
# The monopole (2)

Courtesy: Prof. J. Bartolic, university of Zagreb

Since the radiated power of a monopole fed by the same current is equal to half the radiated power of a dipole, the directivity of a unipole is twice that of a dipole, i.e.

$$D = 3{,}28$$




In dB:

$$10 \log D \approx 5{,}15\ \text{dB}$$

From the definition the effective height of the quarter-wave monopole becomes:

$$h_{ef} = \frac{\lambda}{2\pi} \approx \frac{\lambda}{6}$$

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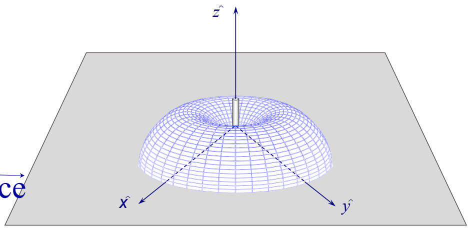
# Radiation pattern of the monopole antenna(3)

3D pattern (ideal)

infinite perfectly conducting surface




**The monopole radiates only in the upper half space !**
Hence its directivity is twice that of a dipole.



Courtesy: Prof. J. Bartolic, university of Zagreb



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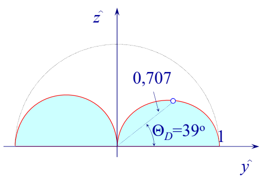
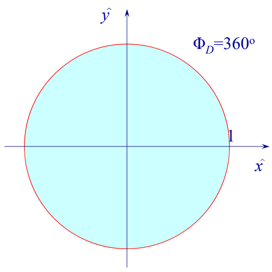
# Radiation pattern of a monopole (4)

2D pattern

*xz and yz planes*




*xy plane*






Courtesy: Prof. J. Bartolic, university of Zagreb

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# Feeding the antenna

- Unbalanced antennas (signal to ground mode)
- Balanced antennas (differential mode)
- Potential problem
- The balun



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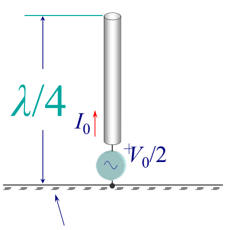
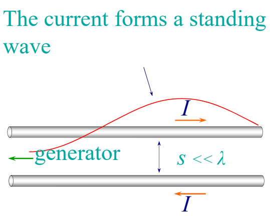
# Unbalanced antenna




The antenna is connected to a signal to ground type (unbalanced, for instance a coaxial cable) transmission line

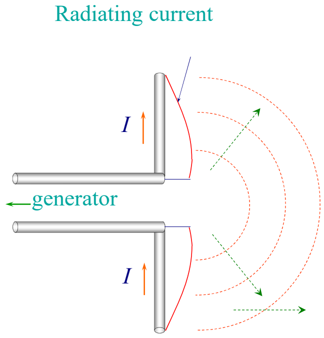
Courtesy: Prof. J. Bartolic, university of Zagreb

# Balanced antenna




Open circuited transmission line

($s$ << $\lambda$) no radiation




Dipole antenna connected to a transmission line

The antenna is connected to a differential type (+/-) transmission line

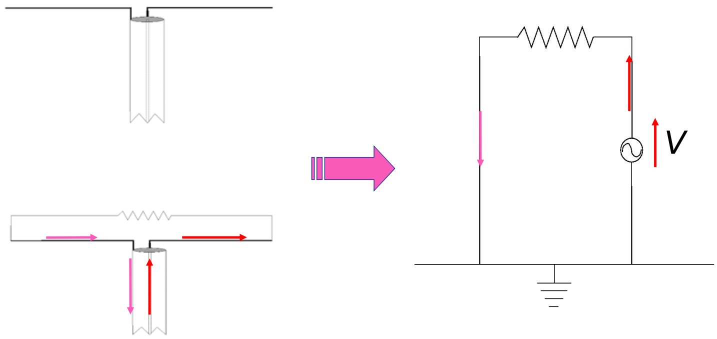
Courtesy: Prof. J. Bartolic, university of Zagreb



# What is the problem ?

- A real life example: a commercial DECT antenna :
  - Ceramic chip, 6 x 9 x 1.8 mm
  - Gain of 2.2dBi at 1.89 GHz
  - Max Gain after Harrington : -3.3 dBi !!
  - Gain measured at LEMA : -8 ± 2 dBi
- The discrepancy is due to measurement errors

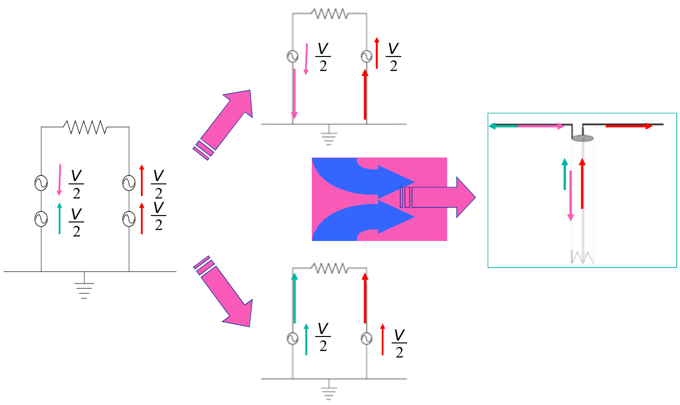
# Spurious radiation from cables



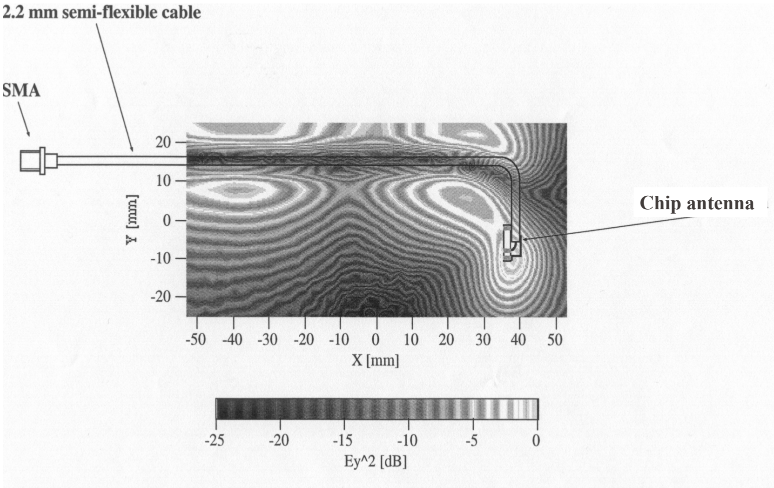





# Spurious radiation from cables

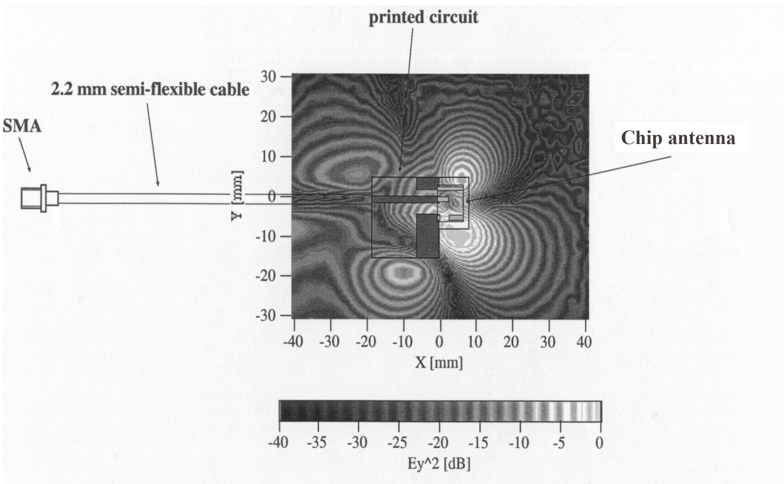


# Effect of Spurious radiation in the case of the chip antenna






# Effect of Spurious radiation in the case of the chip antenna



## Effects on radiation characteristics

- Unwanted radiation in unwanted directions
- Increase of measured gain up to 10 dB
- Destruction of both polarization and radiation pattern

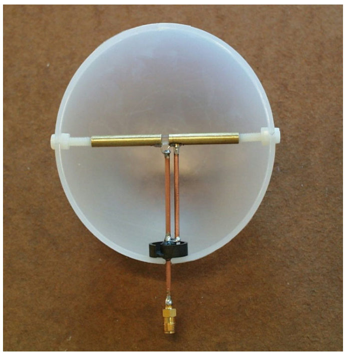


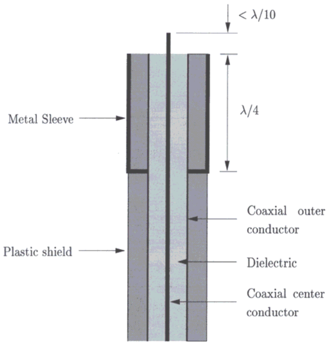

## Measurement solutions

- Baluns
- Wheeler cap method
- System measurement methods
  - reverberation chamber
  - anechoic chamber

# Baluns

The spurious radiation can be attenuated using for instance ferrite cores, chockes or baluns.



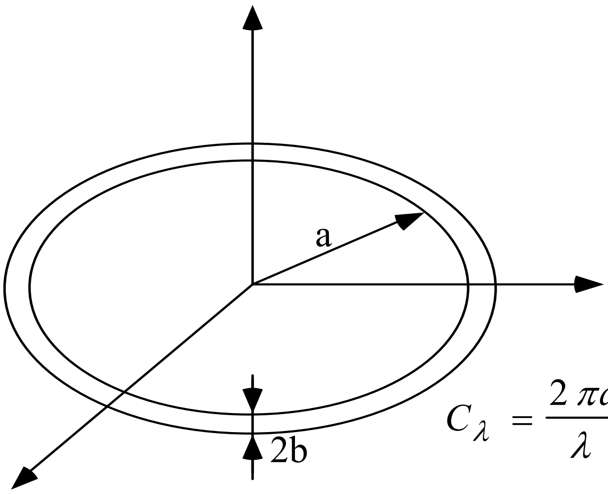





# Baluns

- Advantage :
  - good for both circuit and radiation measurements
- Disadvantages :
  - mostly narrow-band
  - cumbersome for the characterization of multi-band antennas

# The small loop antenna



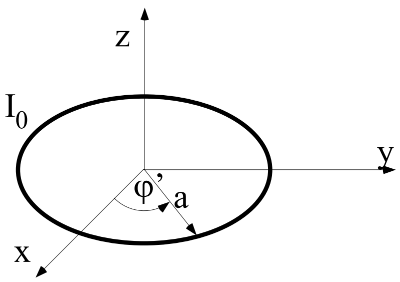

$$C_\lambda = \frac{2\pi a}{\lambda}$$



# Examples



# The small loop antenna




$r' = a \;,\; \mathbf{r'} = a\,\mathbf{e}_\rho$

$\theta' = \dfrac{\pi}{2}$

$I(r',\theta',\varphi') = I(a,\pi/2,\varphi') = I_\varphi = I_0$

The vector integral becomes

$$\boldsymbol{f}(\theta,\varphi) = \int_v dv' \boldsymbol{J}(\boldsymbol{r}') e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}$$

$$\boldsymbol{f}(\theta,\varphi) = I_0 a \int_0^{2\pi} \hat{\boldsymbol{\varphi}}' e^{-jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'} d\varphi'$$

with

$$dv' = \Delta s dl' = \Delta s a d\varphi'$$

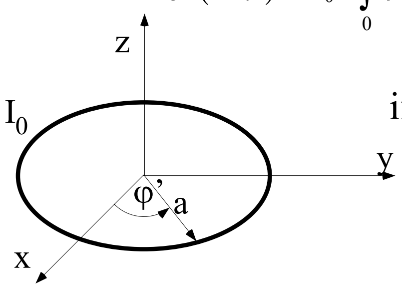
$$\boldsymbol{J} = \frac{\hat{\varphi} I}{\Delta s}$$

$$\hat{\mathbf{r}}\cdot\mathbf{r}' = a\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\rho}}'$$



# The small loop antenna

$$\boldsymbol{f}(\theta,\varphi) = I_0 a \int_0^{2\pi} \hat{\boldsymbol{\varphi}}' e^{-jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'} d\varphi' = I_0 a \int_0^{2\pi} (\cos\varphi'\hat{\boldsymbol{y}} - \sin\varphi'\hat{\boldsymbol{x}}) e^{-jak\hat{\boldsymbol{\rho}}'\cdot\hat{\boldsymbol{r}}} d\varphi'$$




if $ka \ll 1$, $e^{jka\hat{\boldsymbol{\rho}}'\cdot\hat{\boldsymbol{r}}} \approx 1 - jka\hat{\boldsymbol{\rho}}'\cdot\hat{\mathbf{r}}$

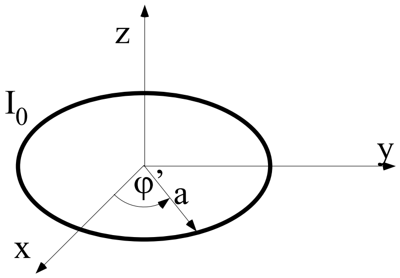
We have a rotational symmetry, so we can limit our study to the x

$$\hat{\boldsymbol{r}} = \cos\theta\hat{\boldsymbol{z}} + \sin\theta\hat{\boldsymbol{x}}$$
$$a\hat{\boldsymbol{\rho}}'\cdot\hat{\boldsymbol{r}} = a\sin\theta\cos\varphi'$$

$$\boldsymbol{f}(\theta,\varphi) = I_0 a \int_0^{2\pi} (\cos\varphi'\hat{\boldsymbol{y}} - \sin\varphi'\hat{\boldsymbol{x}}) e^{-jak\hat{\boldsymbol{\rho}}'\cdot\hat{\boldsymbol{r}}} d\varphi' = I_0 jka^2 \sin\theta\left[\int_0^{2\pi} \cos^2\varphi' d\varphi'\hat{\boldsymbol{y}} - \int_0^{2\pi} \sin\varphi'\cos\varphi' d\varphi'\hat{\boldsymbol{x}}\right]$$

$$= I_0 jka^2 \sin\theta\hat{\boldsymbol{y}}$$

# The small loop antenna




We have a rotational symmetry, we can thus extrapolate what happens in the xz plan to the entire space

$$\boldsymbol{f}(\theta,\varphi) = I_0 jka^2 \sin\theta \hat{\boldsymbol{\varphi}}$$



# Point of interest: the magnetic dipole

- We define an "anti-world", where
  - There exist magnetic charges and currents
  - There are no electric charges and currents



# Maxwell's equations

**real**

$$-\nabla \times \mathbf{E} = j\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\varepsilon\mathbf{E}$$
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$-\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0$$
$$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$$

Existing electrical charges current and surface currents($\rho$, **J**, $\mathbf{J}_s$)

**anti-world**

$$-\nabla \times \mathbf{E} = \mathbf{M} + j\omega\mu\mathbf{H}$$
$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$$
$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = \frac{\rho_m}{\mu}$$
$$-\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = \mathbf{M}_s$$
$$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0$$

Existing magnetic charges current and surface currents($\rho_m$, **M**, $\mathbf{M}_s$)



# Units

$$\rho_m : [Am]$$
$$M : [V/m^2]$$
$$M_s : [V/m]$$
$$M_l : [V]$$

## EPFL Duality principal

| | | | | | | |
|---|---|---|---|---|---|---|
| **J** | ρ | **E** | **H** | ε | μ | Real world |
| **M** | $\rho_m$ | **H** | **-E** | μ | ε | Anti-world |

| | | | | | | |
|---|---|---|---|---|---|---|
| f | λ | k | Zc | I | **A** | Real world |
| f | λ | k | Yc | Im | **Am** | Anti-world |



## EPFL Duality principle

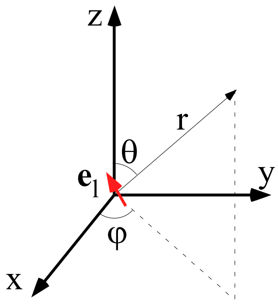
We can define $\boldsymbol{A}_m$ as

$$\boldsymbol{A}_m(\boldsymbol{r}) = \frac{\varepsilon}{4\pi}\frac{\mathrm{e}^{-jkr}}{r}\boldsymbol{f}_m(\theta,\varphi) \text{ with } \boldsymbol{f}_m(\theta,\varphi) = \int_V dv'\boldsymbol{M}(\boldsymbol{r}')e^{jk\hat{\boldsymbol{r}}\cdot\boldsymbol{r}'}$$

And the fields are given by

$$-\boldsymbol{E} = j\omega Z_c \boldsymbol{A_m} \times \hat{\boldsymbol{r}} \quad H = j\omega \hat{\boldsymbol{r}} \times (\hat{\boldsymbol{r}} \times \boldsymbol{A}_m)$$

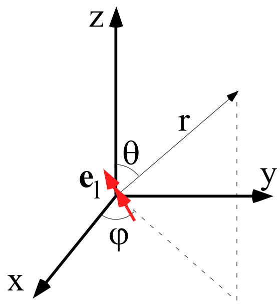
# Example: the magnetic dipole




$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu I \Delta l}{4\pi}\frac{e^{-jkr}}{r}\hat{\boldsymbol{l}}$$

$$\boldsymbol{E}(r,\theta,\varphi) = \frac{jZ_c}{2\lambda} I\Delta l \frac{e^{-jkr}}{r}\hat{\boldsymbol{r}}\times\left(\hat{\boldsymbol{r}}\times\hat{\boldsymbol{l}}\right)$$

$$\boldsymbol{H}(r,\theta,\varphi) = \frac{j}{2\lambda} I\Delta l \frac{e^{-jkr}}{r}\left(\hat{\boldsymbol{l}}\times\hat{\boldsymbol{r}}\right)$$




$$\boldsymbol{A}_m(\boldsymbol{r}) = \frac{\varepsilon I_m \Delta l}{4\pi}\frac{e^{-jkr}}{r}\hat{\boldsymbol{l}}$$

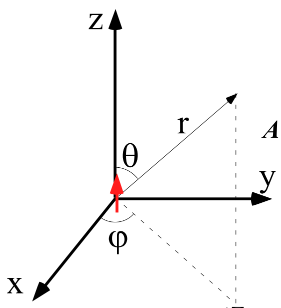
$$\boldsymbol{H}(r,\theta,\varphi) = \frac{j}{2\lambda Z_c} I_m\Delta l \frac{e^{-jkr}}{r}\hat{\boldsymbol{r}}\times\left(\hat{\boldsymbol{r}}\times\hat{\boldsymbol{l}}\right)$$

$$-\boldsymbol{E}(r,\theta,\varphi) = \frac{j}{2\lambda} I_m\Delta l \frac{e^{-jkr}}{r}\left(\hat{\boldsymbol{l}}\times\hat{\boldsymbol{r}}\right)$$



# Example: the magnetic dipole




$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu I \Delta l}{4\pi}\frac{e^{-jkr}}{r}\hat{\boldsymbol{z}}$$

$$E_\theta(r,\theta,\varphi) = \frac{jZ_c}{2\lambda} I\Delta l \frac{e^{-jkr}}{r}\sin\theta$$

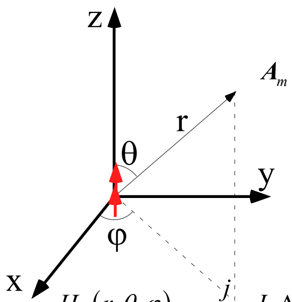
$$E_\varphi(r,\theta,\varphi) = 0$$

$$H_\theta(r,\theta,\varphi) = 0$$

$$H_\varphi(r,\theta,\varphi) = \frac{j}{2\lambda} I\Delta l \frac{e^{-jkr}}{r}\sin\theta$$

$$U(\theta,\varphi) = \frac{Z_c}{8\lambda^2}(I\Delta l)^2\sin^2\theta$$

$$p(r,\theta,\varphi) = \frac{1}{r^2}\frac{Z_c}{8\lambda^2}(I\Delta l)^2\sin^2\theta$$




$$\boldsymbol{A}_m(\boldsymbol{r}) = \frac{\varepsilon I_m \Delta l}{4\pi}\frac{e^{-jkr}}{r}\hat{\boldsymbol{z}}$$

$$H_\theta(r,\theta,\varphi) = \frac{j}{2\lambda Z_c} I_m\Delta l \frac{e^{-jkr}}{r}\sin\theta$$

$$H_\varphi(r,\theta,\varphi) = 0$$

$$-E_\theta(r,\theta,\varphi) = 0$$

$$-E_\varphi(r,\theta,\varphi) = \frac{j}{2\lambda} I_m\Delta l \frac{e^{-jkr}}{r}\sin\theta$$

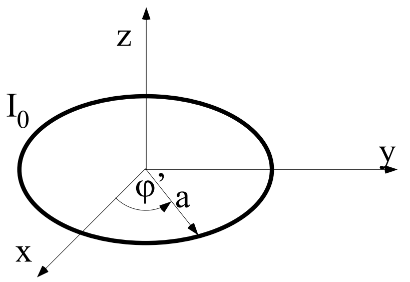
$$U(\theta,\varphi) = \frac{1}{Z_c 8\lambda^2}(I_m\Delta l)^2\sin^2\theta$$

$$p(r,\theta,\varphi) = \frac{1}{8\lambda^2 r^2 Z_c}(I_m\Delta l)^2\sin^2\theta$$



# The small loop



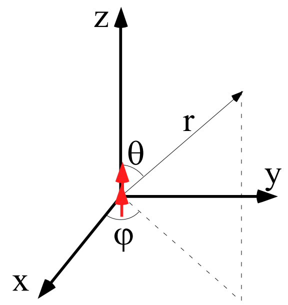

$$H_\theta = -\frac{I_0 (ka)^2 e^{-jkr} \sin\theta}{4r}$$

$$E_\varphi = Z_c \frac{I_0 (ka)^2 e^{-jkr} \sin\theta}{4r}$$

The two are identical for

$$I_m \Delta l = 2\lambda Z_c \frac{I_0 (ka)^2}{4j} = -j \frac{2 I_0 Z_c \pi a^2}{\lambda}$$

# The magnetic dipole




$$H_\theta(r,\theta,\varphi) = \frac{j}{2\lambda Z_c} I_m \Delta l \frac{e^{-jkr}}{r} \sin\theta$$

$$-E_\varphi(r,\theta,\varphi) = \frac{j}{2\lambda} I_m \Delta l \frac{e^{-jkr}}{r} \sin\theta$$



# The small loop antenna

Hence

$$P_{rad} = Z_c \left(\frac{\pi}{24}\right) (ka)^4 |I_0|^2 \qquad R_r = Z_c \left(\frac{\pi}{6}\right) \left(k^2 a^2\right)^2 = 20\pi^2 \left(\frac{C}{\lambda}\right)^4$$

Where *C* is the circumference of the loop

In the case of N windings in the loop:

$$R_r = N^2 Z_c \left(\frac{\pi}{6}\right) \left(k^2 a^2\right)^2 = 20\pi^2 N^2 \left(\frac{C}{\lambda}\right)^4$$

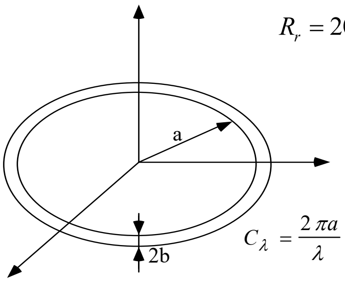
# The small loop

$$U(\theta,\varphi) = \frac{Z_c}{4}\left(\frac{k^2a^2}{4}\right)^2 |I_0|^2 \sin^2\theta$$

$$D_0 = 4\pi\frac{U_{\max}}{P_{rad}} = \frac{3}{2}$$



# The real small loop




$$C_\lambda = \frac{2\pi a}{\lambda}$$

$$R_r = 20\pi^2 C_\lambda^4 \;\; ; \;\; R_{loss} = \frac{a}{b}\sqrt{\frac{\omega\mu_o}{2\sigma}} \;\; ; \;\; L = \mu_o a\left(\ln\frac{8a}{b} - 1.75\right)$$

$$\eta = \frac{R_r}{R_r + R_{loss}}$$

$$\frac{\Delta f}{f_o} = \frac{R_r + R_{loss}}{2\pi f_o L}$$

## Characteristics of a real small loop antenna

Radiation resistance (single turn) $R_r = 20\pi^2 C_\lambda{}^4$

Radiation resistance (N turns) $R_r = 20\pi^2 C_\lambda{}^4 N^2$

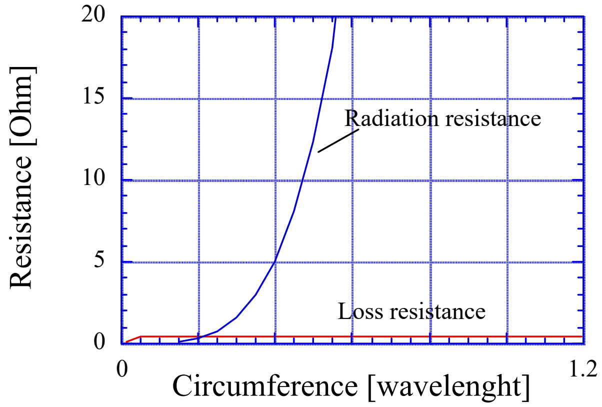
Ohmic loss resistance (single turn) $R_{loss} = \frac{a}{b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$

Ohmic loss resistance (N turns) $R_{loss} = \frac{Na}{b}\sqrt{\frac{\omega\mu_0}{2\sigma}}$

efficiency $\eta = \frac{R_r}{R_r + R_{loss}}$

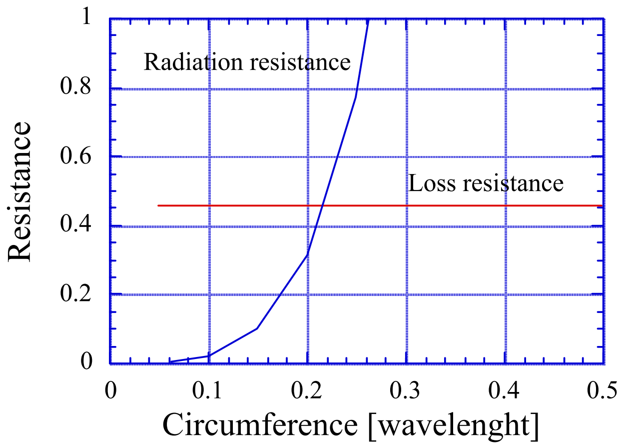


## Single turn loop antenna




Wire: 1mm radius copper, frequency 3 GHz

# Single turn loop antenna




Wire: 1mm radius copper, frequency 3 GHz